Big Ideas Math: Algebra 1

Parent Newsletter

Chapter 11: Rational Equations and Functions

Students will...

- Identify direct and inverse variation.
- Write and graph direct and inverse variation equations.
- Graph rational functions.
- Identify asymptotes.
- Compare graphs of rational functions.
- Find inverse functions.
- Simplify rational expressions.
- Multiply and divide rational expressions.
- Divide polynomials by monomials.
- Divide polynomials by binomials.
- Add and subtract rational expressions.
- Find least common denominators of two rational expressions.
- Solve rational equations using cross products.
- Solve rational equations using least common denominators.
- Solve real-life problems.

Standards

Common Core:
A.REI.10,
A.SSE.2,
A.CED.1, F.BF.4a

Essential Questions

- How can you recognize when two variables vary directly? inversely?
- What are the characteristics of the graph of a rational function?
- How can you simplify a rational expression?
- What are the excluded values of a rational expression?
- How can you multiply and divide rational expressions?
- How can you divide one polynomial by another polynomial?
- How can you add and subtract rational expressions?
- How can you solve a rational equation?

A rational expression is in ***simplest form*** when the numerator and denominator have no common factors except 1.

The least common multiple of the denominators or two or more rational expressions is the ***least common denominator (LCD)*** of the expressions.

A ***rational equation*** is an equation that contains rational expressions.

Key Terms

Two quantities x and y show ***direct variation*** when $y = kx$, where k is a nonzero constant.

Two quantities x and y show ***inverse variation*** when $y = \frac{k}{x}$, where k is a nonzero constant.

A ***rational function*** is a function of the form $y = \frac{\text{polynomial}}{\text{polynomial}}$, where the denominator does not equal 0.

A number that makes a rational function undefined is an ***excluded value***.

Reference Tools

An Example and Non-Example Chart can be used to list examples and non-examples of a vocabulary word or term. Students write examples of the word or term in the left column and non-examples in the right column. This type of organizer serves as a good tool for assessing students' knowledge of pairs of topics that have subtle but important differences.

Inverse Variation Equations

Examples	Non-Examples
$y = \frac{2}{x}$	$y = 2x$
$2 = xy$	$2 = \frac{y}{x}$
$x = \frac{2}{y}$	$y = \frac{x}{2}$
$3xy = 6$	$y = 2x + 1$

An ***asymptote*** is a line that a graph approaches, but never intersects.

An ***inverse relation*** switches the input and output values of the original relation.

When a relation and its inverse are functions, they are called ***inverse functions***.

A ***rational expression*** is an expression that can be written as a fraction whose numerator and denominator are polynomials.

Key Ideas

Direct Variation

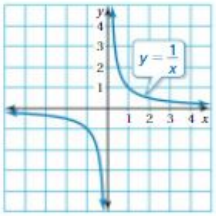
Two quantities x and y show direct variation when $y = kx$, where k is a nonzero constant.

Inverse Variation

Two quantities x and y show direct variation when $y = \frac{k}{x}$, where k is a nonzero constant.

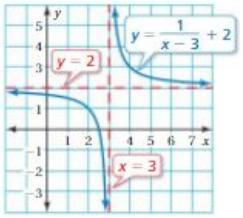
Rational Function

A rational function is a function of the form $y = \frac{\text{polynomial}}{\text{polynomial}}$, where the denominator does not equal 0. The most basic rational function is $y = \frac{1}{x}$.



Asymptotes

The graph of a rational function of the form $y = \frac{a}{x - h} + k$, where $a \neq 0$, has a vertical asymptote $x = h$ and a horizontal asymptote $y = k$.



Simplifying Rational Expressions

- A rational expression is in simplest form when the numerator and denominator have no common factors except 1. To simplify a rational expression, factor the numerator and denominator and divide out any common factors.
- Let a, b, and c be polynomials, where $b, c \neq 0$.

$$\frac{ac}{bc} = \frac{a \times \cancel{c}}{b \times \cancel{c}} = \frac{a}{b}$$

- $\frac{2\cancel{(x + 1)}}{5\cancel{(x + 1)}} = \frac{2}{5}; x \neq -1$

Multiplying and Dividing Rational Expressions

Let a, b, c, and d be polynomials.

- Multiplying: $\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$, where $b, d \neq 0$.
- Dividing: $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c} = \frac{ad}{bc}$, where b, c, and $d \neq 0$.

Adding and Subtracting Rational Expressions with Like Denominators

Let a, b, and c be polynomials, where $c \neq 0$.

- Adding: $\frac{a}{c} + \frac{b}{c} = \frac{a + b}{c}$
- Subtracting: $\frac{a}{c} - \frac{b}{c} = \frac{a - b}{c}$

Quick Review

- The constant k is called the *constant of proportionality* or the *constant of variation*.
- For direct variation equations, you can say "y varies directly with x" or "y is directly proportional to x." For inverse variation functions, you can say "y varies inversely with x" or "y is inversely proportional to x."

What's the Point?

The STEM Videos available online show ways to use mathematics in real-life situations. The Chapter 11: Thunderstorm! STEM Video is available online at www.bigideasmath.com.

- Use the asymptotes to help you draw the ends of a graph.
- The inverse of a function f is written as $f^{-1}(x)$. The -1 in $f^{-1}(x)$ is not an exponent.
- You can see why you can divide out common factors by rewriting the expression. $\frac{ac}{bc} = \frac{a}{b} \cdot \frac{c}{c} = \frac{a}{b} \cdot 1 = \frac{a}{b}$
- Make sure you find excluded values of a rational expression using the original expression.
- When dividing polynomials using long division, first write the polynomials in standard form and insert any missing terms.